Graphs of Equations in Two Variables; Intercepts; Symmetry

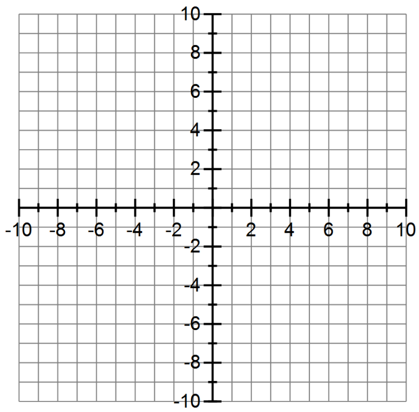
The ***solutions*** of an equation with two variables are pairs of numbers that make the equation true when they are substituted in for the variables.

Graph of an Equation: A drawing that represents all the solutions of an equation. Any ordered pair on the graph is a solution to the equation.

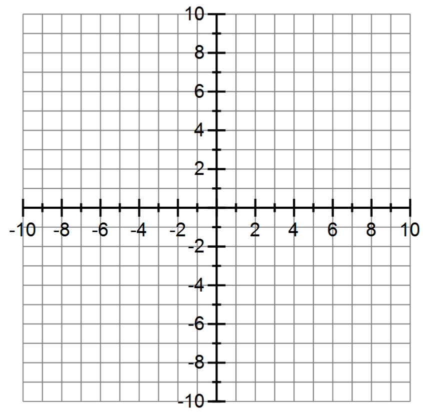
Example: Determine whether the points $(0,0)$, $(1,1)$, and $(1,-1)$ are on the graph of $y = x^3 - 2\sqrt{x}$.

Examples: Graph the following equations by plotting points.

a) $y = 3x - 2$

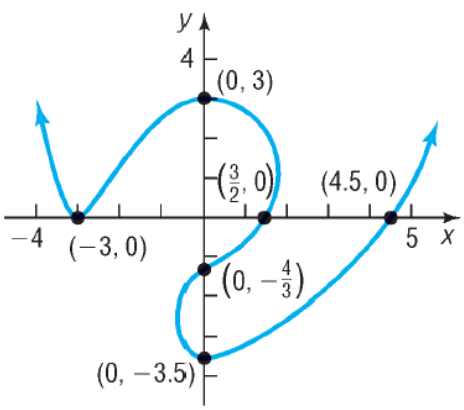


b) $y = x^3$



Intercepts: The points at which a graph crosses or touches the coordinate axes. The x-coordinates of the points where the graph crosses the x-axis are called ***x-intercepts*****.** The y-coordinates of the points where the graph crosses the y-axis are called ***y-intercepts*****.**

Example: Find all the intercepts of the graph.



Notice that at the x-intercepts, y = 0, and at the y-intercepts, x = 0.

Finding Intercepts from an Equation

1. To find the x-intercept(s), let $y = 0$ and solve for x.
2. To find the y-intercept(s), let $x = 0$ and solve for y.

Example: Find the x-intercept(s) and y-intercept(s) of the graphs.

a) $y = x^2 - 6x + 16$

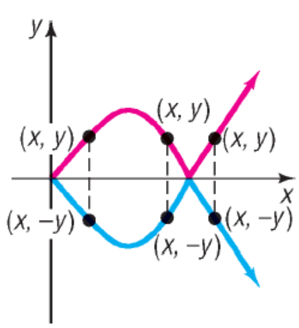
b) $4x^2 + 9y^2 = 36$

Symmetry

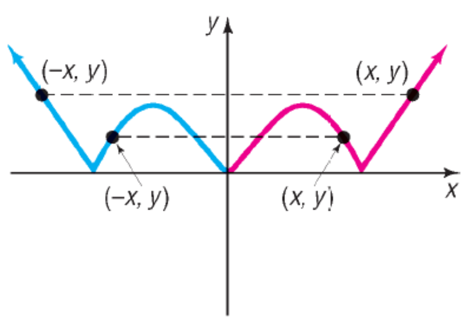
1. A graph is said to be ***symmetric with respect to the x-axis*** if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph. Test by replacing y by $-y$ in the equation and simplifying. If an equivalent equation results, the graph of the equation is symmetric with respect to the x-axis.

2. A graph is said to be ***symmetric with respect to the y-axis*** if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph. Test by replacing x by $-x$ in the equation and simplifying. If an equivalent equation results, the graph of the equation is symmetric with respect to the x-axis.

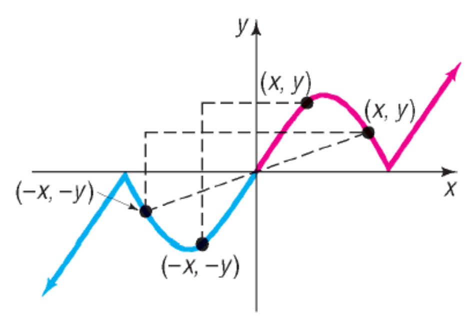
3. A graph is said to be ***symmetric with respect to the origin*** if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph. Test by replacing x by $-x$ and y by $-y$ in the equation and simplifying. If an equivalent equation results, the graph of the equation is symmetric with respect to the origin.



Symmetry with respect to the x-axis



Symmetry with respect to the y-axis



Symmetry with respect to the origin

Examples: Test each equation for symmetry.

a) $y = \dfrac{x^2 - 4}{2x}$

b) $y = \sqrt[5]{x}$

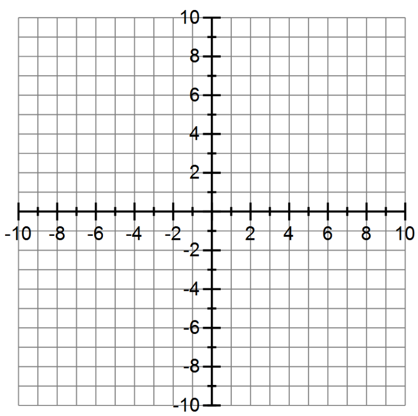
c) $4x + y^2 = 4$

d) $9x^2 + 4y^2 = 36$

Graphing Key Equations

1. Find x-intercepts and y-intercepts.
2. Check for symmetry with respect to the x-axis, the y-axis, and the origin.
3. Create a T-table based on the results of step 1 & 2.

a) $y = 1/x$



b) $x = y^2$